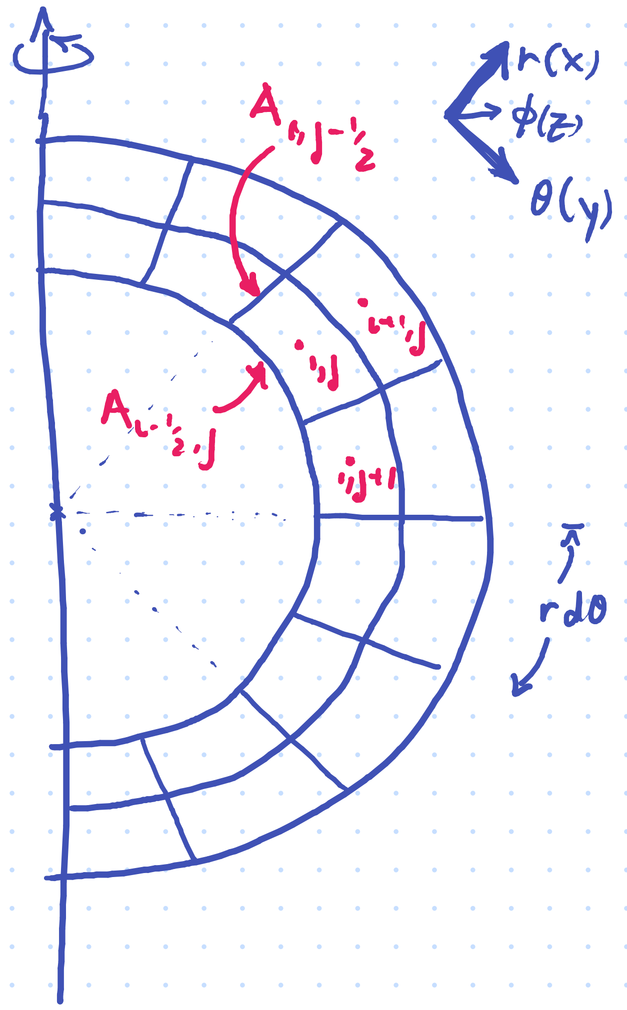

$$V_{i,j} = 2\pi \int_{\theta_l}^{\theta_r} \sin\theta \int_{r_l}^{r_r} r^2 dr$$

$$= \frac{2\pi}{3}(\cos\theta_l - \cos\theta_r)(r_r^3 - r_l^3)$$

Note for $\theta_l = 0$, $\theta_r = \pi$, this gives $V_{i,j} = \frac{4\pi}{3}(r_r^3 - r_l^3)$ as expected

$$A_{i-\frac{1}{2},j} = 2\pi r_l^2 \int_{\theta_l}^{\theta_r} \sin\theta\, d\theta$$

$$= 2\pi r_l^2 (\cos\theta_l - \cos\theta_r)$$

Note for $\theta_l = 0$, $\theta_r = \pi$, this gives $4\pi r_l^2$ as expected

$$A_{i,j-\frac{1}{2}} = 2\pi \sin\theta_l \int_{r_l}^{r_r} dr$$

$$= \pi \sin\theta_l (r_r^2 - r_l^2)$$

Note for $\theta_l = 0$ or π, this gives 0, as expected

For $\theta_l = \frac{\pi}{2}$, this gives $\pi(r_r^2 - r_l^2)$ as expected