Quiz, Hourly Examination-Sem-II (2022-23): STAT-567-Regression Analysis-Cr (1+1)

Time: 2-Hours	Max. Marks: 25 [10 + 15]

Note: *Please solve the following questions.* ***Please do not use lm() model unless allowed for****.*

Problem # 1.

In an experiment with ice growth rates and densities, the growth of the crystals with time is observed. The 43 sets of measurements presented below in Table 1, "Ice Crystallization with Time" are of the mass of the crystals (***M***) in nano-grams for times (***T***) of 50-180 seconds from the introduction of the crystals, where R1, R2 and R3 columns represent the replicated data for the given time (***T***). Each measurement represents a single complete experiment; the experiments were conducted over a number of days and were randomized according as the observation time. It was desired to connect the response ***M*** to the predictor ***T*** by a simple fitted relationship.

Table 1. Ice Crystallization with Time

Time (T)	Crystal Mass (M)		
	R1	R2	R3
50	11.5		
60	8.2	11.5	
70	14.1	17.2	
80	33.5	28.8	
90	15.6	24.4	33.5
95	38.8		
100	47.7	58	36.1
105	47.7	65.5	
110	58	47.7	33.5
115	69.5	69.5	47.7
115	69.5	69.5	47.7
120	87.2	51	33.5
125	47.7		
130	92	87.2	
135	58	47.7	
140	73.2	58	
145	47.7		
150	118.9	58	
155	143.9	87.2	
160	143.9	73.2	73.7
165	97		
170	112.3		
180	113.2		

Perform the following Analyses: (10)

1. Enter the data of Table 1, “Ice Crystallization with Time” in a suitable format as required by analyses. (2)

A possibility to fit the relationship $E(M) = \alpha T^{\beta}$ was suggested. Fit a transformed model $lnM = \alpha + \beta . lnT$, and construct ANOVA table to carry out *F*-test, using formulae approach, and not by ***lm()*** model. Draw conclusion of this ***F***-test. (4)

3. Carry out the analysis of variance for this model and ***test the significance of lack of fit***, if any, for this fitted model. (4)

Problem # 2.

Please refer to the Table 1, "Ice Crystallization with Time". For the data of this table, carry out the following analyses: (15)

1. A possibility to fit the relationship $E(M) = \alpha T^{\beta}$ was suggested. So, estimate the ***Box-Cox transformation index*** λ for fitting the regression model $V = \gamma + \beta . lnT$, where $V = (M^{\lambda} - 1)/(\lambda \dot{M}^{\lambda - 1})$ for $\lambda \neq 0$ and $V = \dot{M} . lnM$ for $\lambda = 0$ with usual meaning of notations. (5)
2. Carry out the analysis of variance of the multiple linear regression model, $V = \gamma + \beta . lnT$ after using the ***Box-Cox transformation index*** and so transforming the the crystal mass (***M***) to $V = (M^{\lambda} - 1)/(\lambda \dot{M}^{\lambda - 1})$ if $\lambda \neq 0$ and $V = \dot{M} . lnM$ if $\lambda = 0$. Write your conclusions of ***F***-test (5)
3. Carry out the ***t***-test of model coefficients γ and β and write their conclusions. (2.5)
4. Draw diagnostic plots using ***lm()*** model, and write your overall conclusions from these diagnostic plots. (2.5)